Access to Science, Engineering and Agriculture: Mathematics 1
MATH00030
Chapter 4 Exercises

1. For each of the following:

   (i) Say whether or not it is a function and if not say why not.

   (ii) If it is a function state the domain and the codomain.

   (a)

   $$f \colon \mathbb{R} \to \mathbb{R}^+$$
   $$x \mapsto x$$

   (b)

   $$f \colon \mathbb{R}^+ \to \mathbb{R}$$
   $$x \mapsto x$$

   (c)

   $$f \colon \mathbb{R}^+ \to \mathbb{R}$$
   $$x \mapsto -x^2$$

   (d)

   $$f \colon \mathbb{R} \to \mathbb{R}^+$$
   $$x \mapsto -x^2$$

   (e) In the following, $\mathbb{R}^-$ will denote the non-positive real numbers.

   $$f \colon \mathbb{R}^- \to \mathbb{R}^+$$
   $$x \mapsto \sqrt{-x}$$

   (f)

   $$f \colon \mathbb{R}^+ \to \mathbb{R}^-$$
   $$x \mapsto \sqrt{x}$$

   (g)

   $$f \colon \mathbb{R}^+ \to \mathbb{R}^+$$
   $$x \mapsto x - 1$$

2. Sketch the graph of each of the functions in Question 1 (including arrows on the end of the curves if necessary).

3. Sketch the graph of each of the following functions.

(a)
$$f\colon \{-36, -25, -16, -9, -4, 0\} \to \mathbb{R}^+$$
$$x \mapsto \sqrt{-x}$$

(b)
$$f\colon \{-4, -2, 0, 1, 4\} \to \{0, 2, 4\}$$
$$-4 \mapsto 0$$
$$-2 \mapsto 4$$
$$0 \mapsto 2$$
$$1 \mapsto 2$$
$$4 \mapsto 4$$

(c)
$$f\colon \{x \in \mathbb{R}\colon\ -1 \leqslant x \leqslant 2\} \to \{x \in \mathbb{R}\colon\ -1 \leqslant x \leqslant 7\}$$
$$x \mapsto 2x + 1$$

4. For each of the following functions say whether they are injective, surjective or bijective. If a function is not injective or surjective then say why not.

(a)
$$f\colon \{A, B, C, D, E\} \to \{1, 2, 3, 4\}$$
$$A \mapsto 2$$
$$B \mapsto 2$$
$$C \mapsto 1$$
$$D \mapsto 4$$
$$E \mapsto 3$$

(b)
$$f\colon \{A, B, C, D, E\} \to \{1, 2, 3, 4\}$$
$$A \mapsto 1$$
$$B \mapsto 3$$
$$C \mapsto 1$$
$$D \mapsto 3$$
$$E \mapsto 4$$

(c)

$$f\colon \{A, B, C, D\} \to \{1, 2, 3, 4, 5\}$$
$$A \mapsto 1$$
$$B \mapsto 2$$
$$C \mapsto 2$$
$$D \mapsto 4$$

(d)

$$f\colon \{A, B, C, D\} \to \{1, 2, 3, 4, 5\}$$
$$A \mapsto 3$$
$$B \mapsto 4$$
$$C \mapsto 5$$
$$D \mapsto 2$$

(e)

$$f\colon \{A, B, C, D\} \to \{1, 2, 3, 4\}$$
$$A \mapsto 2$$
$$B \mapsto 3$$
$$C \mapsto 1$$
$$D \mapsto 1$$

(f)

$$f\colon \{A, B, C, D\} \to \{1, 2, 3, 4\}$$
$$A \mapsto 2$$
$$B \mapsto 3$$
$$C \mapsto 1$$
$$D \mapsto 4$$

(g)

$$f\colon \mathbb{R} \to \mathbb{R}$$
$$x \mapsto 3x - 4$$

(h)

$$f\colon \mathbb{R}^- \to \mathbb{R}$$
$$x \mapsto 3x - 4$$

(i)

$$f\colon \mathbb{R}^+ \to \mathbb{R}$$
$$x \mapsto 3x - 4$$

(j)

$$f \colon \mathbb{R}^- \to \mathbb{R}$$
$$x \mapsto -2x^2$$

(k)

$$f \colon \mathbb{R} \to \mathbb{R}^-$$
$$x \mapsto -2x^2$$

(l)

$$f \colon \mathbb{R}^- \to \mathbb{R}^-$$
$$x \mapsto -2x^2$$

5. For each of the bijective functions you found in Question 4, give the inverse function.

6. Figure 1 contains the graphs of four of the following functions:

   (i) $y = 3^x$.
   (ii) $y = -4^x$.
   (iii) $y = -\left(\dfrac{2}{3}\right)^x$.
   (iv) $y = \left(\dfrac{3}{4}\right)^x$.
   (v) $y = \log_3(x)$.
   (vi) $y = \log_{1/4}(x)$.
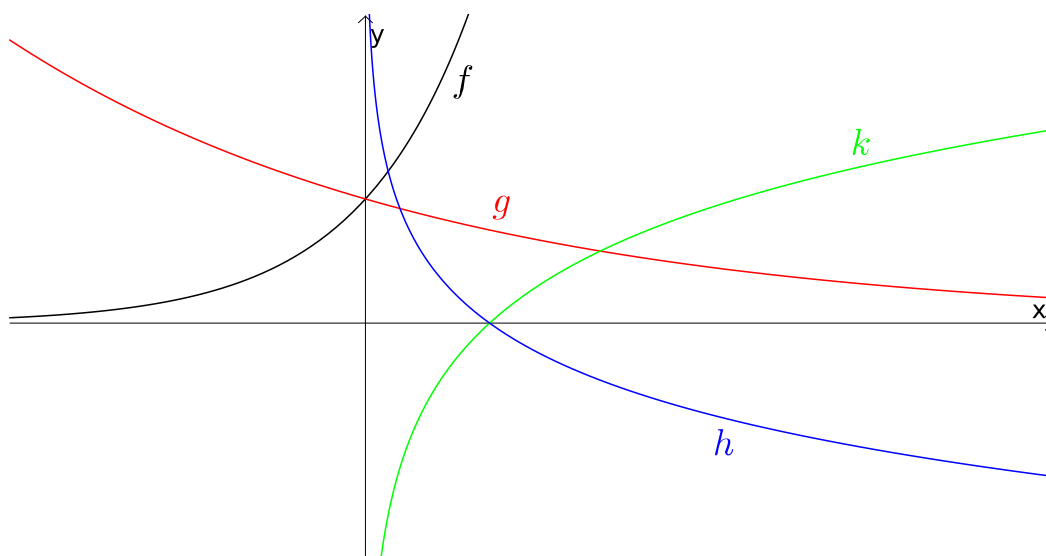
   Match the functions to the graphs.

Figure 1: Various functions

7. For each of the following equations, either solve it or show it has no solutions.

(a) $e^x = 5$.

(b) $4^x = 7$.

(c) $-7^{3x} = 5$.

(d) $10^{7x} = 3$.

(e) $9^{2x} = 8$.

(f) $e^{-5x} = 4$.

(g) $3^{-6x} = 2$.

(h) $-9^{-5x} = 7$.

(i) $5\left(10^{-3x}\right) = 6$.

(j) $-7\left(8^{-7x}\right) = -4$.

(k) $-6\left(5^{-2x}\right) = 5$.

8. (a) Convert the following angles to radians, leaving your answers as multiples of π.
   (i) $30°$.
   (ii) $135°$.
   (iii) $150°$.
   (iv) $330°$.

(b) Convert the following angles to degrees.
   (i) $\dfrac{\pi}{4}$ Radians.
   (ii) $\dfrac{\pi}{2}$ Radians.
   (iii) $\dfrac{2\pi}{3}$ Radians.
   (iv) $\dfrac{7\pi}{4}$ Radians.

9. Without using a calculator, find the following:

(a) $\sin\left(\dfrac{5\pi}{6}\right)$.

(b) $\cos\left(\dfrac{7\pi}{6}\right)$.

(c) $\tan\left(\dfrac{5\pi}{4}\right)$.

(d) $\sin\left(-\dfrac{\pi}{4}\right)$.

(e) $\operatorname{cosec}\left(\dfrac{4\pi}{3}\right)$.

(f) $\cot\left(\dfrac{9\pi}{4}\right)$.

(g) $\cos\left(\dfrac{38\pi}{3}\right)$.